

# Entanglement detection via general SIC-POVMs

Ya Xi and Zhu-Jun Zheng

Department of Mathematics, South China University of Technology, GuangZhou 510640, China

We study the quantum separability problem by using general symmetric informationally complete measurements and derive separability criteria for arbitrary high dimensional bipartite systems of a  $d_1$ -dimensional subsystem and a  $d_2$ -dimensional subsystem and multipartite systems of multipartite-level subsystems. These criteria are of more effective and wider application range than previous criteria. They provide experimental implementation in detecting entanglement of unknown quantum states.

PACS numbers: 03.67.Mn, 03.67.Hk

## I. INTRODUCTION

The detection of entanglement is one of the most fundamental and attractive tasks in quantum information theory and quantum information processing. It is well-known that quantum entanglement enables numerous applications ranging from quantum cryptography to quantum computing (see reviews [1, 2] and the references therein). For pure quantum states, there have been numerous criteria to distinguish quantum entangled states from the separable ones. For the general mixed states, there has been considerable effort to analyze the separability. But till now we don't have an operational criterion of separability. There have been some necessary criterion for separability, such as Bell inequality [3], positive partial transposition criterion [4, 5, 8], realignment criterion [9–13], covariance matrix criterion [14], and correlation matrix criterion [15–17], entanglement witness [5–7].

Although numerous mathematical tools have been employed in entanglement detection of given known quantum states, experimental implementation of entanglement detection for unknown quantum states has fewer results [18–21]. The authors [22] connected the separability criteria to mutually unbiased bases (MUBs) [23] in two-qudit, multipartite and continuous-variable quantum systems. Based on the correlation functions, the criterion uses local measurements only, and can be implemented experimentally. In Ref. [24], Chen *et al.* generalized such idea and proposed a separability criteria for two-qudit states by using mutually unbiased measurements (MUMs) [25]. It is shown that the criterion based on MUMs is more effective than the criterion based on MUBs and for isotropic states this criterion becomes both necessary and sufficient. After that, Liu *et al.* [26] derived separability criteria for arbitrary high-dimensional bipartite and multipartite systems using sets of MUMs.

Besides mutually unbiased bases, another intriguing topic in quantum information theorem is the symmetric informationally complete positive operator-valued measurements (SIC-POVMs) [27, 28]. Most of the literature on SIC-POVMs focus on rank 1 SIC-POVMs (all the POVM elements are proportional to rank 1 projectors). Such rank 1 SIC-POVMs have been shown analytically

to exist in dimensions  $d = 1, \dots, 16, 19, 24, 28, 35, 48$ , and numerically for all dimensions  $d \leq 67$  (see [28] and references therein). However, despite the enormous effort of the last years, it is still not known if rank 1 SIC-POVMs exist in all finite dimensions. In [29], the author introduced the concept of general SIC-POVMs in which the elements need not to be of rank one, and showed that such general SIC-POVMs exist in all finite dimensions. And then Gour and Kalev [30] constructed the set of all general SIC-POVMs from the generalized Gell-Mann matrices. Recently, in Ref. [31], the authors used the general SIC-POVMs to derive separability criteria for arbitrary  $d$ -dimensional bipartite and multipartite systems.

In this paper, we study separability problem via general SIC-POVMs and propose some criteria for the separability of arbitrary high dimensional bipartite systems and multipartite systems of multi-lever subsystems.

## II. SICS AND GENERAL SICS

A POVM  $\{P_j\}$  with  $d^2$  rank one operators acting on  $\mathbb{C}^d$  is symmetric informationally complete, if

$$(1) P_j = \frac{1}{d} |\phi_j\rangle\langle\phi_j|, j = 1, 2, \dots, d^2,$$

$$(2) \sum_{j=1}^d P_j = I$$

where the vectors  $|\phi_j\rangle$  satisfy  $|\langle\phi_j|\phi_k\rangle|^2 = \frac{1}{d+1}, j \neq k$ , and  $I$  is the identity matrix. The existence of SIC-POVMs in arbitrary dimension  $d$  is an open problem. Only in a number of low dimensional cases, the existence of SIC-POVMs has been proved analytically, and numerically for all dimensions up to 67 (see [28] and the references therein).

A set of  $d^2$  positive-semidefinite operators  $\{P_\alpha\}_{\alpha=1}^{d^2}$  on  $\mathbb{C}_d$  is said to be a general SIC measurement, if

$$(1) \sum_{\alpha=1}^{d^2} P_\alpha = I,$$

$$(2) \text{Tr}[(P_\alpha)^2] = a,$$

$$(3) \text{Tr}(P_\alpha P_\beta) = \frac{1-da}{d(d^2-1)}$$

where  $\alpha, \beta = 1, 2, \dots, d^2, \alpha \neq \beta, I$  is the identity operator, and the parameter  $a$  satisfies  $\frac{1}{d^3} < a \leq \frac{1}{d^2}$ . Moreover

$a = \frac{1}{d^2}$  if and only if  $P_\alpha$  are rank one, which gives rise to a SIC-POVM.

Moreover, like the mutually unbiased measurements, in Ref. [29, 30], the authors explicitly constructed general symmetric informationally complete measurements for arbitrary dimensional spaces. Let  $\{F_\alpha\}_{\alpha=1}^{d^2-1}$  be an orthonormal basis of a real vector space of dimensional  $d^2 - 1$ , satisfying  $\text{Tr}(F_\alpha F_\beta) = \delta_{\alpha,\beta}$ ,  $\alpha, \beta = 1, 2, \dots, d^2 - 1$ . Define  $F = \sum_{\alpha=1}^{d^2-1} F_\alpha$ , then the  $d^2$  operators

$$(1) P_\alpha = \frac{1}{d^2} I + t[F - d(d+1)F_\alpha], \alpha = 1, 2, \dots, d^2 - 1,$$

$$(2) P_{d^2} = \frac{1}{d^2} I + t(d+1)F,$$

form a general SIC-POVM measurement. Here  $t$  should be chosen such that  $P_\alpha \geq 0$ , and corresponding to the construction of general SIC-POVMs, the parameter  $a$  is given by

$$a = \frac{1}{d^3} + t^2(d-1)(d+1)^3.$$

The entanglement detection based SIC-POVMs has been briefly discussed in Ref. [32], but the method is subject to the existence of SIC-POVMs. However these general symmetric informationally complete measurements do exist for arbitrary dimension  $d$ , and have many useful applications in quantum information theory. And in Ref. [33], based on the calculation of the so-called index of the coincidence, the author derived a number of uncertainty relation inequalities via general SIC-POVMs measurements. In addition, for the given SIC-POVM  $\mathcal{P} = \{P_j\}$  on  $\mathbb{C}^d$  and the density matrix  $\rho$ , the author [33] proposed a equality about the index of the coincidence  $\mathcal{C}(\mathcal{P}|\rho)$ , that is,

$$\mathcal{C}(\mathcal{P}|\rho) = \frac{(ad^3 - 1)\text{Tr}(\rho^2) + d(1 - ad)}{d(d^2 - 1)} \quad (1)$$

where

$$\mathcal{C}(\mathcal{P}|\rho) = \sum_{j=1}^{d^2} [\text{Tr}(P_j \rho)]^2$$

At the same time, when the density matrix  $\rho$  is pure,  $\mathcal{C}(\mathcal{P}|\rho) = \frac{ad^2+1}{d(d+1)}$ .

### III. GENERAL SIC-POVMS BASED SEPARABILITY CRITERION

In Ref. [31], the authors used the general SIC-POVMs to propose separability criteria for arbitrary  $d$ -dimensional bipartite and multipartite systems. Here we will give the criterion about arbitrary high dimensional bipartite and multipartite systems.

**Theorem 1** Suppose  $\rho$  is a density matrix in  $\mathbb{C}^{d_1} \otimes \mathbb{C}^{d_2}$ . Let  $\mathcal{P} = \{P_j\}_{j=1}^{d_1^2}$  and  $\mathcal{Q} = \{Q_k\}_{k=1}^{d_2^2}$  be any two sets

of general SIC-POVMs on  $\mathbb{C}^{d_1}$  and  $\mathbb{C}^{d_2}$  with parameters  $a_1, a_2$ , respectively. Define

$$J(\rho) = \max_{\substack{\{P_{n_j}\} \subseteq \{P_j\} \\ \{Q_{n_j}\} \subseteq \{Q_j\}}} \sum_{j=1}^d \text{Tr}[(P_{n_j} \otimes Q_{n_j})\rho]$$

Here  $d = \min\{(d_1)^2, (d_2)^2\}$ . If  $\rho$  is separable, then

$$J(\rho) \leq \frac{1}{2} \left[ \frac{a_1 d_1^2 + 1}{d_1(d_1 + 1)} + \frac{a_2 d_2^2 + 1}{d_2(d_2 + 1)} \right]$$

[Proof]. Assume that  $\rho = \sum_k p_k \rho_k$ ,  $\sum_k p_k = 1$ ,  $I(\rho_k) = \sum_{j=1}^d \text{Tr}[(P_{n_j} \otimes Q_{n_j})\rho_k]$ , where  $\rho_k = |\phi\rangle\langle\phi| \otimes |\psi\rangle\langle\psi|$ . From the linearity of the trace function, we need only to consider pure separable states  $\rho_k$ . So we have

$$\begin{aligned} I(\rho_k) &= \sum_{j=1}^d \text{Tr}(P_{n_j} \otimes Q_{n_j} \rho_k) \\ &= \sum_{j=1}^d \text{Tr}(P_{n_j} |\phi\rangle\langle\phi|) \text{Tr}(Q_{n_j} |\psi\rangle\langle\psi|) \\ &\leq \frac{1}{2} \sum_{j=1}^d \{[\text{Tr}(P_{n_j} |\phi\rangle\langle\phi|)]^2 + [\text{Tr}(Q_{n_j} |\psi\rangle\langle\psi|)]^2\} \\ &\leq \frac{1}{2} \left[ \frac{a_1 d_1^2 + 1}{d_1(d_1 + 1)} + \frac{a_2 d_2^2 + 1}{d_2(d_2 + 1)} \right]. \end{aligned}$$

Then we can get

$$\begin{aligned} J(\rho) &= \max_{\substack{\{P_{n_j}\} \subseteq \{P_j\} \\ \{Q_{n_j}\} \subseteq \{Q_j\}}} \sum_{j=1}^d \text{Tr}[(P_{n_j} \otimes Q_{n_j})\rho] \\ &= \max_k \sum_k p_k I(\rho_k) \\ &\leq \frac{1}{2} \left[ \frac{a_1 d_1^2 + 1}{d_1(d_1 + 1)} + \frac{a_2 d_2^2 + 1}{d_2(d_2 + 1)} \right] \end{aligned}$$

So  $J(\rho) \leq \frac{1}{2} \left[ \frac{a_1 d_1^2 + 1}{d_1(d_1 + 1)} + \frac{a_2 d_2^2 + 1}{d_2(d_2 + 1)} \right]$ .  $\square$

It is worthy to note that the criteria in Ref. [31] is the corollary of Theorem 1. In fact, if  $d_1 = d_2 = d$ , and  $\{P_j\}_{j=1}^{d^2}$  and  $\{Q_k\}_{k=1}^{d^2}$  are any two sets of general SIC-POVMs on  $\mathbb{C}_d$ , with the same parameter  $a$ , then by Theorem 1 there is

$$J(\rho) = \max_{\substack{\{P_{n_j}\} \subseteq \{P_j\} \\ \{Q_{n_j}\} \subseteq \{Q_j\}}} \sum_{j=1}^{d^2} \text{Tr}[(P_{n_j} \otimes Q_{n_j})\rho] \leq \frac{ad^2 + 1}{d(d+1)},$$

which is the desired result. Therefore, the criterion in Ref. [31] is the special case of our criterion of Theorem 1. And Our criteria is also both necessary and sufficient

for the separability of isotropic states. Unlike the criterion based on SIC-POVMs in Ref. [32], our criteria work perfectly for any dimensional  $d$ .

By using the Cauchy-Schwarz inequality, we can obtain stronger bound than in Theorem 1.

**Theorem 2** Let  $\rho$  be a density matrix in  $\mathbb{C}^{d_1} \otimes \mathbb{C}^{d_2}$ ,  $\mathcal{P} = \{P_j\}_{j=1}^{d_1^2}$  and  $\mathcal{Q} = \{Q_j\}_{j=1}^{d_2^2}$  be any two sets of general SIC-POVMs on  $\mathbb{C}^{d_1}$  and  $\mathbb{C}^{d_2}$  with parameters  $a_1, a_2$ . Define

$$J(\rho) = \max_{\substack{\{P_{n_j}\} \subseteq \{P_j\} \\ \{Q_{n_j}\} \subseteq \{Q_j\}}} \sum_{j=1}^d \text{Tr}[(P_{n_j} \otimes Q_{n_j})\rho]$$

Here  $d = \min\{(d_1)^2, (d_2)^2\}$ , If  $\rho$  is separable, then

$$J(\rho) \leq \sqrt{\frac{a_1 d_1^2 + 1}{d_1(d_1 + 1)}} \sqrt{\frac{a_2 d_2^2 + 1}{d_2(d_2 + 1)}}.$$

[Proof]. Let  $I(\rho) = \sum_{j=1}^d \text{Tr}[(P_{n_j} \otimes Q_{n_j})\rho]$ . As theorem 1, we need only to consider pure separable state  $\rho = |\phi\rangle\langle\phi| \otimes |\psi\rangle\langle\psi|$ , since  $\text{Tr}[(P_{n_j} \otimes Q_{n_j})\rho]$  is a linear function. Then we have

$$\begin{aligned} I(\rho) &= \sum_{j=1}^d \text{Tr}[(P_{n_j} \otimes Q_{n_j})\rho] \\ &= \sum_{j=1}^d [\text{Tr}(P_{n_j} |\phi\rangle\langle\phi|)] [\text{Tr}(Q_{n_j} |\psi\rangle\langle\psi|)] \\ &\leq \sqrt{\sum_{j=1}^d [\text{Tr}(P_{n_j} |\phi\rangle\langle\phi|)]^2} \sqrt{\sum_{j=1}^d [\text{Tr}(Q_{n_j} |\psi\rangle\langle\psi|)]^2} \\ &\leq \sqrt{\sum_{j=1}^d [\text{Tr}(P_{n_j} |\phi\rangle\langle\phi|)]^2} \sqrt{\sum_{j=1}^d [\text{Tr}(Q_{n_j} |\psi\rangle\langle\psi|)]^2} \\ &= \sqrt{\frac{a_1 d_1^2 + 1}{d_1(d_1 + 1)}} \sqrt{\frac{a_2 d_2^2 + 1}{d_2(d_2 + 1)}} \end{aligned}$$

Here we use the Cauchy-Schwarz inequality and the equality (1). Then for any density matrix, we can have

$$J(\rho) \leq \sqrt{\frac{a_1 d_1^2 + 1}{d_1(d_1 + 1)}} \sqrt{\frac{a_2 d_2^2 + 1}{d_2(d_2 + 1)}}. \square$$

The bound in Theorem 2 is lower than that in Theorem 1 since  $\sqrt{\frac{a_1 d_1^2 + 1}{d_1(d_1 + 1)}} \sqrt{\frac{a_2 d_2^2 + 1}{d_2(d_2 + 1)}} \leq \frac{1}{2} [\frac{a_1 d_1^2 + 1}{d_1(d_1 + 1)} + \frac{a_2 d_2^2 + 1}{d_2(d_2 + 1)}]$ .

It is characteristic for multipartite systems that the definition of entanglement is not unique. For the reason, we can discuss it with the notions of  $k$ -partite entanglement or  $k$ -nonseparability for given partition and unfixed partition, respectively [1, 2]. A pure state  $|\phi\rangle$  of a  $n$ -partite system is called  $k$ -separable if it can be written as a tensor product of  $k$  vectors, i.e.  $|\phi\rangle = |\phi\rangle_1 \otimes |\phi\rangle_2 \otimes \cdots \otimes |\phi\rangle_k$ . States that are  $n$ -separable do

not contain any entanglement and are called fully separable. In addition, those states whose entanglement ranges over all  $n$  parties are called genuine multipartite entangled states. The generalization to mixed states is direct: A mixed state is called  $k$ -separable if it can be written as a convex combination of  $k$ -separable states  $\rho = \sum_k p_k \rho_k$ , where  $\rho_k$  is  $k$ -separable pure states. In the following, we will give two criteria for multipartite systems and then argue  $k$ -nonseparability for a given partition of  $n$ -partite system.

**Theorem 3** Suppose  $\rho$  is a density matrix in  $\mathbb{C}^{d_1} \otimes \mathbb{C}^{d_2} \otimes \cdots \otimes \mathbb{C}^{d_m}$ , and  $\{\mathcal{P}^{(i)}\}$  are  $m$  sets of general SIC-POVMs on  $\mathbb{C}^{d_i}$  with parameters  $a_i, i = 1, 2, \dots, m$ , where  $\{\mathcal{P}^{(i)}\} = \{P_j^{(i)}\}_{j=1}^{d_i^2}$ . Define

$$J(\rho) = \max_{\{P_{n_j}^{(i)}\} \subseteq \{\mathcal{P}^{(i)}\}} \sum_{j=1}^d \text{Tr}(\bigotimes_{i=1}^m P_{n_j}^{(i)} \rho)$$

Here  $d = \min\{(d_1)^2, (d_2)^2, \dots, (d_m)^2\}$ . If  $\rho$  is fully separable, then

$$J(\rho) \leq \frac{1}{m} \sum_{i=1}^m \left[ \frac{a_i d_i^2 + 1}{d_i(d_i + 1)} \right]$$

[Proof]. Let  $\rho = \sum_k p_k P_k$ , with  $\sum_k p_k = 1$ , be a fully separable density matrix, where  $\rho_k = \bigotimes_{i=1}^m |\phi_{ik}\rangle\langle\phi_{ik}|$ . Since

$$\begin{aligned} \sum_{j=1}^d \text{Tr}(\bigotimes_{i=1}^m P_{n_j}^{(i)} \rho_k) &= \sum_{j=1}^d \text{Tr}[(\bigotimes_{i=1}^m P_{n_j}^{(i)}) (\bigotimes_{i=1}^m |\phi_{ik}\rangle\langle\phi_{ik}|)] \\ &= \sum_{j=1}^d [\prod_{i=1}^m \text{Tr}(P_{n_j}^{(i)} |\phi_{ik}\rangle\langle\phi_{ik}|)] \\ &\leq \sum_{j=1}^d \left\{ \frac{\sum_{i=1}^m [\text{Tr}(P_{n_j}^{(i)} |\phi_{ik}\rangle\langle\phi_{ik}|)]^2}{m} \right\}^{\frac{m}{2}} \\ &\leq \sum_{j=1}^d \frac{\sum_{i=1}^m [\text{Tr}(P_{n_j}^{(i)} |\phi_{ik}\rangle\langle\phi_{ik}|)]^2}{m} \\ &= \sum_{i=1}^m \sum_{j=1}^d \frac{[\text{Tr}(P_{n_j}^{(i)} |\phi_{ik}\rangle\langle\phi_{ik}|)]^2}{m} \\ &\leq \frac{1}{m} \sum_{i=1}^m \left[ \frac{a_i d_i^2 + 1}{d_i(d_i + 1)} \right] \end{aligned}$$

where we use the inequality  $x_1 x_2 \cdots x_n \leq \left[ \frac{\sum_{i=1}^n (x_i)^2}{n} \right]^{\frac{n}{2}}$ , where  $x_i \geq 0, i = 1, 2, \dots, n$  [26]

Then

$$\begin{aligned} \sum_{j=1}^d \text{Tr}(\bigotimes_{i=1}^m P_{n_j}^{(i)} \rho) &= \sum_k \sum_{j=1}^d p_k \text{Tr}(\bigotimes_{i=1}^m P_{n_j}^{(i)} \rho_k) \\ &\leq \frac{1}{m} \sum_{i=1}^m \left[ \frac{a_i d_i^2 + 1}{d_i(d_i + 1)} \right] \end{aligned}$$

Finally, we can get  $J(\rho) \leq \frac{1}{m} \sum_{i=1}^m \left[ \frac{a_i d_i^2 + 1}{d_i(d_i + 1)} \right]$ .  $\square$

**Theorem 4** Assume that  $\rho$  is a density matrix in  $\mathbb{C}^{d_1} \otimes \mathbb{C}^{d_2} \otimes \dots \otimes \mathbb{C}^{d_m}$ , and  $\{P_j^{(i)}\}_{j=1}^{(d_i)^2}$  are  $m$  sets of general SIC-POVMs on  $\mathbb{C}^{d_i}$  with parameters  $a_i, i = 1, 2, \dots, m$ . Define

$$J(\rho) = \max_{\{P_{n_j}^{(i)}\} \subseteq \{\mathcal{P}_j^{(i)}\}} \sum_{j=1}^d \text{Tr}(\bigotimes_{i=1}^m P_{n_j}^{(i)} \rho)$$

Here  $d = \min\{(d_1)^2, (d_2)^2, \dots, (d_m)^2\}$ . If  $\rho$  is fully separable, then

$$J(\rho) \leq \min_{i \neq j} \sqrt{\frac{a_i d_i^2 + 1}{d_i(d_i + 1)}} \sqrt{\frac{a_j d_j^2 + 1}{d_j(d_j + 1)}}$$

[Proof]. Let  $\rho = \sum_k p_k P_k$  be a fully separable pure state, where  $\sum_k p_k = 1$ .

$$\begin{aligned} I(\rho) &= \sum_{j=1}^d \sum_k p_k \text{Tr}[(\bigotimes_{i=1}^m P_{n_j}^{(i)}) P_k] \\ &= \sum_k \sum_{j=1}^d p_k \text{Tr}[(\bigotimes_{i=1}^m P_{n_j}^{(i)}) (\bigotimes_{i=1}^m |\phi_i\rangle\langle\phi_i|)] \\ &= \sum_k \sum_{j=1}^d p_k \text{Tr}[\bigotimes_{i=1}^m (P_{n_j}^{(i)} |\phi_i\rangle\langle\phi_i|)] \\ &= \sum_k \sum_{j=1}^d p_k \prod_{i=1}^m \text{Tr}(P_{n_j}^{(i)} |\phi_i\rangle\langle\phi_i|) \\ &\leq \sum_k \sum_{j=1}^d p_k \text{Tr}(P_{n_j}^{(i)} |\phi_i\rangle\langle\phi_i|) \text{Tr}(P_{n_j}^{(i')} |\phi_{i'}\rangle\langle\phi_{i'}|) \end{aligned}$$

Then using the Cauchy-Schwarz inequality, we can get

$$I(\rho) \leq \sqrt{\sum_{j=1}^d [\text{Tr}(P_{n_j}^{(i)} |\phi_i\rangle\langle\phi_i|)]^2} \sqrt{\sum_{j=1}^d [\text{Tr}(P_{n_j}^{(i')} |\phi_{i'}\rangle\langle\phi_{i'}|)]^2}$$

So using the equality (1), we finally get  $J(\rho) = \max I(\rho) \leq \min_{i \neq j} \sqrt{\frac{a_i d_i^2 + 1}{d_i(d_i + 1)}} \sqrt{\frac{a_j d_j^2 + 1}{d_j(d_j + 1)}}$ .  $\square$

For Theorem 3 and Theorem 4, we don't require the subsystems with the same dimension, so we can use them

straightforward to detect  $k$ -nonseparable states with respect to a fixed partition. For an  $n$ -partite state  $\rho$  in  $\mathbb{C}^1 \otimes \mathbb{C}^2 \otimes \dots \otimes \mathbb{C}^n = \mathbb{C}^{d_1} \otimes \mathbb{C}^{d_2} \otimes \dots \otimes \mathbb{C}^{d_k}$ , if there are sets of  $k$  general SIC-POVMs  $\{\mathcal{P}^{(i)}\}$  on  $\mathbb{C}^{d_i}$  with parameters  $a_i$  such that

$$\sum_{j=1}^{d^2} \text{Tr}(\bigotimes_{i=1}^k P_{n_j}^{(i)} \rho) > \frac{1}{k} \sum_{i=1}^k \frac{a_i d_i^2 + 1}{d_i(d_i + 1)}$$

or

$$\sum_{j=1}^{d^2} \text{Tr}(\bigotimes_{i=1}^k P_{n_j}^{(i)} \rho) > \min_{1 \leq i \neq j \leq k} \sqrt{\frac{a_i d_i^2 + 1}{d_i(d_i + 1)}} \sqrt{\frac{a_j d_j^2 + 1}{d_j(d_j + 1)}}$$

for some  $\{P_{n_j}^{(i)}\}_{j=1}^{d^2} \subseteq \{\mathcal{P}^{(i)}\}$ , then  $\rho$  is  $k$ -nonseparable in  $\mathbb{C}^{d_1} \otimes \mathbb{C}^{d_2} \otimes \dots \otimes \mathbb{C}^{d_k}$ , where  $d = \min\{d_1, d_2, \dots, d_k\}$  and  $i = 1, 2, \dots, k$ .

Our criteria are better than the previous ones [22, 24, 26, 32]. First, Comparing with [22] and [32], our criteria are suitable for arbitrary dimension  $d$  because the general SIC-POVMs do exist for arbitrary dimension  $d$ . Second, we can detect entanglement more wider and effective than the authors [24] and [26]. The criteria in this paper could detect the separability of arbitrary high dimensional bipartite systems and multipartite systems of different dimensions, but the criteria [24] are not. To Ref. [26], we just need less joint local measurements, reducing the complexity of experimental implementation.

#### IV. CONCLUSION AND DISCUSSIONS

We have studied the separability problem via general SIC-POVMs and have presented separability criteria for the separability of arbitrary high dimensional bipartite systems of a  $d_1$ -dimensional subsystem and a  $d_2$ -dimensional subsystem and multipartite systems of multipartite-level subsystems. For isotropic states, our criteria are both necessary and sufficient. It detects all the entangled isotropic states of arbitrary dimension  $d$ . The powerfulness of our criterion is due to that these criteria are more effective and wider applications range than previous criteria and comparing with the criteria based on mutually unbiased measurements, our criteria require less local measurements. For multipartite systems, we can detect the  $k$ -nonseparability of  $n$ -partite and high dimensional systems. It would be interesting to study the separability criteria of multipartite systems with different dimensions via complete set of general SIC-POVMs.

**Acknowledgments** This work is supported by the NSFC through Grants No.11475178 and No.11571119. And we would like to thank Prof. Shao-Ming Fei at School of Mathematical Sciences, Capital Normal University for helpful discussion.

- 
- [1] R. Horodecki, P. Horodecki, M. Horodecki, and K. Horodecki, *Rev. Mod. Phys.* **81**, 865 (2009).
  - [2] O. Guhne, G. Toth, *Phys. Rep.* **474**, 1 (2009).
  - [3] J.S. Bell, *Physics(N.Y.)* **1**, 195 (1964).
  - [4] A. Peres, *Phys. Rev. Lett.* **77**, 1413 (1996).
  - [5] M. Horodecki, P. Horodecki, and R. Horodecki, *Phys. Lett. A* **223**, 1 (1996).
  - [6] B. Terhal, *Phys. Lett. A* **271**, 319 (2000).
  - [7] M. Lewenstein, B. Kraus, J.I. Cirac and P. Horodecki, *Phys. Rev. A* **62**, 052310 (2000).
  - [8] P. Horodecki, *Phys. Lett. A* **232**, 333 (1997).
  - [9] O. Rudolph, *Phys. Rev. A* **67**, 032312 (2003).
  - [10] K. Chen and L. A. Wu, *Quant. Inf. Comput.* **3**, 193 (2003).
  - [11] M. Horodecki, P. Horodecki, and R. Horodecki, *Open Syst. Inf. Dyn.* **13**, 103 (2006).
  - [12] K. Chen and L. A. Wu, *Phys. Lett. A* **306**, 14 (2002); *Phys. Rev. A* **69**, 022312 (2004); P. Wocjan and M. Horodecki, *Open Syst. Inf. Dyn.* **12**, 331 (2005).
  - [13] S. Albeverio, K. Chen, and S. M. Fei, *Phys. Rev. A* **68**, 062313 (2003).
  - [14] O. Guhne, P. Hyllus, O. Gittsovich, and J. Eisert, *Phys. Rev. Lett.* **99**, 130504 (2007).
  - [15] J. D. Vicente, *Quant. Inf. Comput.* **7**, 624 (2007).
  - [16] J. D. Vicente, *J. Phys. A: Math. Theor.* **41**, 065309 (2008).
  - [17] M. Li, J. Wang, S.-M. Fei, and X. Li-Jost, *Phys. Rev. A* **89**, 022325 (2014).
  - [18] N. Gisin, *Phys. Lett. A* **154**, 201 (1991).
  - [19] S. Yu, J.W. Pan, Z.B. Chen and Y.D. Zhang, *Phys. Rev. Lett.* **91**, 217903 (2003).
  - [20] M. Li and S.M. Fei, *Phys. Rev. Lett.* **104**, 240502 (2010).
  - [21] M.J. Zhao, T. Ma, S.M. Fei and Z.X. Wang, *Phys. Rev. A* **83**, 052120 (2011).
  - [22] C. Spengler, M. Huber, S. Brierley, T. Adaktylos, and B. C. Hiesmayr, *Phys. Rev. A* **86**, 022311 (2012).
  - [23] W. K. Wootters and B. D. Fields, *Ann. Phys. (N.Y.)* **191**, 363 (1989).
  - [24] B. Chen, T. Ma, and S.M. Fei, *Phys. Rev. A* **89**, 064302 (2014).
  - [25] A. Kalev and G. Gour, *New J.Phys.* **16**, 053038 (2014).
  - [26] L. Liu, T. Gao, and F. I. Yan, arXiv: 1501.01717[quant-ph] (2015).
  - [27] J. M. Renes, R. Blume-Kohout, A. J. Scott, C. M. Caves, *J. Math. Phys.* **45**, 2171 (2004).
  - [28] A. J. Scott, M. Grassl, *J. Math. Phys.* **51**, 042203 (2010).
  - [29] D. M. Appleby, *Optics and Spectroscopy* **103**, 416 (2007).
  - [30] G. Gour and A. Kalev, *J. Phys. A: Math. Theor* **47**, 335302 (2014).
  - [31] B. Chen, Tao. Li, and S.M. Fei, *Quantum. information. processing* **14**, 2281-2290 (2015).
  - [32] A. E. Rastegin, *Eur. Phys. J. D* **67**, 269 (2013).
  - [33] A. E. Rastegin, *Phys. Scr.* **89**, 085101 (2014).